[10-09-08-T12]

Choosing delta

■ Prove $\lim_{x \to 3} (x^2 + x - 5) = 7$

Preliminary analysis (finding a suitable δ)

$|(x^2 + x - 5) - 7| < \epsilon$

$\Longleftrightarrow |x^2 + x - 12| < \epsilon$

$\Longleftrightarrow |x + 4||x - 3| < \epsilon$

Now, suppose we make $|x - 3| < \delta = 1$. Then $-1 < x - 3 < 1 \Longrightarrow 6 < x + 4 < 8 \Longrightarrow |x + 4| < 8$.

As long as $\delta \leq 1$, $|x + 4||x - 3| < 8|x - 3|$. Now, $8|x - 3| < \epsilon$ when $|x - 3| < \frac{\epsilon}{8}$.

So,

$|x + 4||x - 3| < \epsilon \Longleftrightarrow |x + 4| < 8$ and $|x - 3| < \frac{\epsilon}{8}$, providing $\delta \leq 1$.

Notice that the choice $\delta = \frac{\epsilon}{8}$ will, if it is less than 1, make $|x + 4||x - 3| < \epsilon$. If $\frac{\epsilon}{8}$ is greater than 1, then the choice $\delta = 1$ will do the job.

We can guarantee that for every $\epsilon > 0$, $|x + 4||x - 3| < \epsilon$ by choosing $\delta = \min\{1, \frac{\epsilon}{8}\}$.

Do note that the preliminary analysis is not a part of the proof. It need not be a piece of rigorous mathematics. However one arrives at the correct choice of δ is fine. The strategies demonstrated in the text and in class are effective in finding the correct choice of δ for any ϵ and that is why you are learning them.